Arbeitsblatt

06.12.2020

Kostenlos auf dw-aufgaben.de

Aufgaben-Quickname: 1000

Aufgabe 1

Quick: 1000

Löse das Gleichungssystem.

a)
$$\begin{array}{rcrcrcrcr} 10x_1 & + & 8x_2 & + & -2x_3 & + & 4x_4 & = & 122 \\ 60x_1 & + & 50x_2 & + & -5x_3 & + & 23x_4 & = & 761 \\ -40x_1 & + & -44x_2 & + & -38x_3 & + & -18x_4 & = & -690 \\ 70x_1 & + & 40x_2 & + & -34x_3 & + & 117x_4 & = & 892 \end{array}$$

Variablenwerte herleiten:

$(4) \Rightarrow x_4 = 2$
$(3) \Rightarrow (-4)x_3 + (-16) = (-28) \Rightarrow (-4)x_3 = -12 \Rightarrow x_3 = 3$
$(2) \Rightarrow 2x_2 + 21 + (-2) = 29 \Rightarrow 2x_2 = 10 \Rightarrow x_2 = 5$
$(1) \Rightarrow 10x_1 + 40 + (-6) + 8 = 122 \Rightarrow 10x_1 = 80 \Rightarrow x_1 = 8$

Lösung: $x_1 = 8, x_2 = 5, x_3 = 3, x_4 = 2$

b)
$$\begin{array}{rcrcrcrcr} -8x_1 & + & 8x_2 & + & -5x_3 & + & -6x_4 & = & 94 \\ 16x_1 & + & -8x_2 & + & 16x_3 & + & 17x_4 & = & -185 \\ -8x_1 & + & 24x_2 & + & 2x_3 & + & 5x_4 & = & 139 \\ 8x_1 & & & + & 11x_3 & + & 4x_4 & = & -84 \end{array}$$

Variablenwerte herleiten:

$(4) \Rightarrow x_4 = -1$
$(3) \Rightarrow (-5)x_3 + (-1) = 39 \Rightarrow (-5)x_3 = 40 \Rightarrow x_3 = -8$
$(2) \Rightarrow 8x_2 + (-48) + (-5) = 3 \Rightarrow 8x_2 = 56 \Rightarrow x_2 = 7$
$(1) \Rightarrow (-8)x_1 + 56 + 40 + 6 = 94 \Rightarrow (-8)x_1 = -8 \Rightarrow x_1 = 1$

Lösung: $x_1 = 1, x_2 = 7, x_3 = -8, x_4 = -1$

Aufgabe 2

Quick: 1000

Löse das Gleichungssystem.

a)
$$\begin{array}{rcrcrcrcr} -2x_1 & + & -5x_2 & + & -8x_3 & + & -4x_4 & = & 12 \\ 6x_1 & + & 18x_2 & + & 22x_3 & + & 17x_4 & = & 46 \\ -12x_1 & + & -24x_2 & + & -60x_3 & + & -18x_4 & = & 276 \\ -16x_1 & + & -49x_2 & + & -50x_3 & + & -52x_4 & = & -262 \end{array}$$

Dreiecksform:

$$\begin{array}{rcrcrcrcrl} -2x_1 & + & -5x_2 & + & -8x_3 & + & -4x_4 & = & 12 & (1) \\ & & 3x_2 & + & -2x_3 & + & 5x_4 & = & 82 & (2) \\ & & & & -8x_3 & + & -4x_4 & = & 40 & (3) \\ & & & & & & -9x_4 & = & -72 & (4) \end{array}$$

Variablenwerte herleiten:

$(4) \Rightarrow x_4 = 8$
$(3) \Rightarrow (-8)x_3 + (-32) = 40 \Rightarrow (-8)x_3 = 72 \Rightarrow x_3 = -9$
$(2) \Rightarrow 3x_2 + 18 + 40 = 82 \Rightarrow 3x_2 = 24 \Rightarrow x_2 = 8$
$(1) \Rightarrow (-2)x_1 + (-40) + 72 + (-32) = 12 \Rightarrow (-2)x_1 = 12 \Rightarrow x_1 = -6$

Lösung: $x_1 = -6, x_2 = 8, x_3 = -9, x_4 = 8$

b)

$$\begin{array}{rcrcrcrcr} 7x_1 & + & -10x_2 & + & 2x_3 & + & 9x_4 & = & 177 \\ -7x_1 & + & 15x_2 & + & 6x_3 & + & -4x_4 & = & -135 \\ -70x_1 & + & 85x_2 & + & -53x_3 & + & -99x_4 & = & -1890 \\ 56x_1 & + & -40x_2 & + & 98x_3 & + & 99x_4 & = & 1733 \end{array}$$

Dreiecksform:

$$\begin{array}{rcrcrcrcrl} 7x_1 & + & -10x_2 & + & 2x_3 & + & 9x_4 & = & 177 & (1) \\ & & 5x_2 & + & 8x_3 & + & 5x_4 & = & 42 & (2) \\ & & & & -9x_3 & + & 6x_4 & = & 6 & (3) \\ & & & & & & -x_4 & = & -7 & (4) \end{array}$$

Variablenwerte herleiten:

$(4) \Rightarrow x_4 = 7$
$(3) \Rightarrow (-9)x_3 + 42 = 6 \Rightarrow (-9)x_3 = -36 \Rightarrow x_3 = 4$
$(2) \Rightarrow 5x_2 + 32 + 35 = 42 \Rightarrow 5x_2 = -25 \Rightarrow x_2 = -5$
$(1) \Rightarrow 7x_1 + 50 + 8 + 63 = 177 \Rightarrow 7x_1 = 56 \Rightarrow x_1 = 8$

Lösung: $x_1 = 8, x_2 = -5, x_3 = 4, x_4 = 7$

Aufgabe 3

Quick: 1000

Löse das Gleichungssystem. Benutze das Gaußsche Eliminationsverfahren.

a)

$$\begin{array}{rcrcrcrcr} 2x_1 & + & -9x_2 & + & -7x_3 & + & -8x_4 & = & 84 \\ -20x_1 & + & 82x_2 & + & 63x_3 & + & 73x_4 & = & -769 \\ -4x_1 & + & 2x_2 & + & 3x_3 & + & 4x_4 & = & -70 \\ -12x_1 & + & 86x_2 & + & 43x_3 & + & 64x_4 & = & -434 \end{array}$$

$$\begin{array}{rcrcrcrcrll} 2x_1 & + & -9x_2 & + & -7x_3 & + & -8x_4 & = & 84 & (1) & \\ -20x_1 & + & 82x_2 & + & 63x_3 & + & 73x_4 & = & -769 & (2) & \mid + 10 \text{ x } (1) \\ -4x_1 & + & 2x_2 & + & 3x_3 & + & 4x_4 & = & -70 & (3) & \\ -12x_1 & + & 86x_2 & + & 43x_3 & + & 64x_4 & = & -434 & (4) & \end{array}$$

$$\begin{array}{rcrcrcrcrll} 2x_1 & + & -9x_2 & + & -7x_3 & + & -8x_4 & = & 84 & (1) & \\ & & -8x_2 & + & -7x_3 & + & -7x_4 & = & 71 & (2) & \\ -4x_1 & + & 2x_2 & + & 3x_3 & + & 4x_4 & = & -70 & (3) & \mid + 2 \text{ x } (1) \\ -12x_1 & + & 86x_2 & + & 43x_3 & + & 64x_4 & = & -434 & (4) & \end{array}$$

$$\begin{array}{rcrcrcrcrll} 2x_1 & + & -9x_2 & + & -7x_3 & + & -8x_4 & = & 84 & (1) & \\ & & -8x_2 & + & -7x_3 & + & -7x_4 & = & 71 & (2) & \\ & & -16x_2 & + & -11x_3 & + & -12x_4 & = & 98 & (3) & \\ -12x_1 & + & 86x_2 & + & 43x_3 & + & 64x_4 & = & -434 & (4) & \mid + 6 \text{ x } (1) \end{array}$$

$$
\begin{array}{rcrcrcrcll}
2x_1 & + & -9x_2 & + & -7x_3 & + & -8x_4 & = & 84 & (1) \\
 & & -8x_2 & + & -7x_3 & + & -7x_4 & = & 71 & (2) \\
 & & -16x_2 & + & -11x_3 & + & -12x_4 & = & 98 & (3) \quad | + (-2) \text{ x } (2) \\
 & & 32x_2 & + & x_3 & + & 16x_4 & = & 70 & (4)
\end{array}
$$

$$
\begin{array}{rcrcrcrcll}
2x_1 & + & -9x_2 & + & -7x_3 & + & -8x_4 & = & 84 & (1) \\
 & & -8x_2 & + & -7x_3 & + & -7x_4 & = & 71 & (2) \\
 & & & & 3x_3 & + & 2x_4 & = & -44 & (3) \\
 & & 32x_2 & + & x_3 & + & 16x_4 & = & 70 & (4) \quad | + 4 \text{ x } (2)
\end{array}
$$

$$
\begin{array}{rcrcrcrcll}
2x_1 & + & -9x_2 & + & -7x_3 & + & -8x_4 & = & 84 & (1) \\
 & & -8x_2 & + & -7x_3 & + & -7x_4 & = & 71 & (2) \\
 & & & & 3x_3 & + & 2x_4 & = & -44 & (3) \\
 & & & & -27x_3 & + & -12x_4 & = & 354 & (4) \quad | + 9 \text{ x } (3)
\end{array}
$$

$$
\begin{array}{rcrcrcrcll}
2x_1 & + & -9x_2 & + & -7x_3 & + & -8x_4 & = & 84 & (1) \\
 & & -8x_2 & + & -7x_3 & + & -7x_4 & = & 71 & (2) \\
 & & & & 3x_3 & + & 2x_4 & = & -44 & (3) \\
 & & & & & & 6x_4 & = & -42 & (4)
\end{array}
$$

Variablenwerte herleiten:

$(4) \Rightarrow x_4 = -7$
$(3) \Rightarrow 3x_3 + (-14) = (-44) \Rightarrow 3x_3 = -30 \Rightarrow x_3 = -10$
$(2) \Rightarrow (-8)x_2 + 70 + 49 = 71 \Rightarrow (-8)x_2 = -48 \Rightarrow x_2 = 6$
$(1) \Rightarrow 2x_1 + (-54) + 70 + 56 = 84 \Rightarrow 2x_1 = 12 \Rightarrow x_1 = 6$

Lösung: $x_1 = 6, x_2 = 6, x_3 = -10, x_4 = -7$

b)

$$
\begin{array}{rcrcrcrcl}
-3x_1 & + & 7x_2 & + & -3x_3 & + & 6x_4 & = & -26 \\
21x_1 & + & -51x_2 & + & 16x_3 & + & -43x_4 & = & 198 \\
30x_1 & + & -72x_2 & + & 31x_3 & + & -65x_4 & = & 272 \\
-6x_1 & + & 32x_2 & + & 81x_3 & + & -9x_4 & = & -220
\end{array}
$$

$$
\begin{array}{rcrcrcrcll}
-3x_1 & + & 7x_2 & + & -3x_3 & + & 6x_4 & = & -26 & (1) \\
21x_1 & + & -51x_2 & + & 16x_3 & + & -43x_4 & = & 198 & (2) \quad | + 7 \text{ x } (1) \\
30x_1 & + & -72x_2 & + & 31x_3 & + & -65x_4 & = & 272 & (3) \\
-6x_1 & + & 32x_2 & + & 81x_3 & + & -9x_4 & = & -220 & (4)
\end{array}
$$

$$
\begin{array}{rcrcrcrcll}
-3x_1 & + & 7x_2 & + & -3x_3 & + & 6x_4 & = & -26 & (1) \\
 & & -2x_2 & + & -5x_3 & + & -x_4 & = & 16 & (2) \\
30x_1 & + & -72x_2 & + & 31x_3 & + & -65x_4 & = & 272 & (3) \quad | + 10 \text{ x } (1) \\
-6x_1 & + & 32x_2 & + & 81x_3 & + & -9x_4 & = & -220 & (4)
\end{array}
$$

$$
\begin{array}{rcrcrcrcll}
-3x_1 & + & 7x_2 & + & -3x_3 & + & 6x_4 & = & -26 & (1) \\
 & & -2x_2 & + & -5x_3 & + & -x_4 & = & 16 & (2) \\
 & & -2x_2 & + & x_3 & + & -5x_4 & = & 12 & (3) \\
-6x_1 & + & 32x_2 & + & 81x_3 & + & -9x_4 & = & -220 & (4) \quad | + (-2) \text{ x } (1)
\end{array}
$$

$$\begin{array}{rcrcrcrcll}
-3x_1 & + & 7x_2 & + & -3x_3 & + & 6x_4 & = & -26 & (1) \\
 & & -2x_2 & + & -5x_3 & + & -x_4 & = & 16 & (2) \\
 & & -2x_2 & + & x_3 & + & -5x_4 & = & 12 & (3) \quad | + (-1) \text{ x } (2) \\
 & & 18x_2 & + & 87x_3 & + & -21x_4 & = & -168 & (4)
\end{array}$$

$$\begin{array}{rcrcrcrcll}
-3x_1 & + & 7x_2 & + & -3x_3 & + & 6x_4 & = & -26 & (1) \\
 & & -2x_2 & + & -5x_3 & + & -x_4 & = & 16 & (2) \\
 & & & & 6x_3 & + & -4x_4 & = & -4 & (3) \\
 & & 18x_2 & + & 87x_3 & + & -21x_4 & = & -168 & (4) \quad | + 9 \text{ x } (2)
\end{array}$$

$$\begin{array}{rcrcrcrcll}
-3x_1 & + & 7x_2 & + & -3x_3 & + & 6x_4 & = & -26 & (1) \\
 & & -2x_2 & + & -5x_3 & + & -x_4 & = & 16 & (2) \\
 & & & & 6x_3 & + & -4x_4 & = & -4 & (3) \\
 & & & & 42x_3 & + & -30x_4 & = & -24 & (4) \quad | + (-7) \text{ x } (3)
\end{array}$$

$$\begin{array}{rcrcrcrcll}
-3x_1 & + & 7x_2 & + & -3x_3 & + & 6x_4 & = & -26 & (1) \\
 & & -2x_2 & + & -5x_3 & + & -x_4 & = & 16 & (2) \\
 & & & & 6x_3 & + & -4x_4 & = & -4 & (3) \\
 & & & & & & -2x_4 & = & 4 & (4)
\end{array}$$

Variablenwerte herleiten:

$(4) \Rightarrow x_4 = -2$
$(3) \Rightarrow 6x_3 + 8 = (-4) \Rightarrow 6x_3 = -12 \Rightarrow x_3 = -2$
$(2) \Rightarrow (-2)x_2 + 10 + 2 = 16 \Rightarrow (-2)x_2 = 4 \Rightarrow x_2 = -2$
$(1) \Rightarrow (-3)x_1 + (-14) + 6 + (-12) = (-26) \Rightarrow (-3)x_1 = -6 \Rightarrow x_1 = 2$

Lösung: $x_1 = 2, x_2 = -2, x_3 = -2, x_4 = -2$

Aufgabe 4

Quick: 1000

Löse das Gleichungssystem. Benutze das Gaußsche Eliminationsverfahren.

a)
$$\begin{array}{rcrcrcrcll}
-3a & + & -10b & + & -c & = & -23 & & & \\
-18a & + & -67b & + & 3c & = & -98 & & & \\
24a & + & 38b & + & 68c & = & 460 & & &
\end{array}$$

$$\begin{array}{rcrcrcrcll}
-3a & + & -10b & + & -c & = & -23 & (1) & & \\
-18a & + & -67b & + & 3c & = & -98 & (2) & | + (-6) \text{ x } (1) & \\
24a & + & 38b & + & 68c & = & 460 & (3) & &
\end{array}$$

$$\begin{array}{rcrcrcrcll}
-3a & + & -10b & + & -c & = & -23 & (1) & & \\
 & & -7b & + & 9c & = & 40 & (2) & & \\
24a & + & 38b & + & 68c & = & 460 & (3) & | + 8 \text{ x } (1) &
\end{array}$$

$$\begin{array}{rcrcrcrcll}
-3a & + & -10b & + & -c & = & -23 & (1) & & \\
 & & -7b & + & 9c & = & 40 & (2) & & \\
 & & -42b & + & 60c & = & 276 & (3) & | + (-6) \text{ x } (2) &
\end{array}$$

$$\begin{array}{rcrcrcrcll}
-3a & + & -10b & + & -c & = & -23 & (1) & & \\
 & & -7b & + & 9c & = & 40 & (2) & & \\
 & & & & 6c & = & 36 & (3) & &
\end{array}$$

Variablenwerte herleiten:

(3) $\Rightarrow x_3 = 6$
(2) $\Rightarrow (-7)x_2 + 54 = 40 \Rightarrow (-7)x_2 = -14 \Rightarrow x_2 = 2$
(1) $\Rightarrow (-3)x_1 + (-20) + (-6) = (-23) \Rightarrow (-3)x_1 = 3 \Rightarrow x_1 = -1$

Lösung: $a = -1, b = 2, c = 6$

b)
$$\begin{array}{rcrcrcl} -2a & + & 7b & + & 7c & = & -66 \\ 10a & + & -40b & + & -31c & = & 371 \\ 12a & + & -77b & + & -12c & = & 681 \end{array}$$

$$\begin{array}{rcrcrcll} -2a & + & 7b & + & 7c & = & -66 & (1) \\ 10a & + & -40b & + & -31c & = & 371 & (2) \quad \mid +5 \text{ x } (1) \\ 12a & + & -77b & + & -12c & = & 681 & (3) \end{array}$$

$$\begin{array}{rcrcrcll} -2a & + & 7b & + & 7c & = & -66 & (1) \\ & & -5b & + & 4c & = & 41 & (2) \\ 12a & + & -77b & + & -12c & = & 681 & (3) \quad \mid +6 \text{ x } (1) \end{array}$$

$$\begin{array}{rcrcrcll} -2a & + & 7b & + & 7c & = & -66 & (1) \\ & & -5b & + & 4c & = & 41 & (2) \\ & & -35b & + & 30c & = & 285 & (3) \quad \mid +(-7) \text{ x } (2) \end{array}$$

$$\begin{array}{rcrcrcll} -2a & + & 7b & + & 7c & = & -66 & (1) \\ & & -5b & + & 4c & = & 41 & (2) \\ & & & & 2c & = & -2 & (3) \end{array}$$

Variablenwerte herleiten:

(3) $\Rightarrow x_3 = -1$
(2) $\Rightarrow (-5)x_2 + (-4) = 41 \Rightarrow (-5)x_2 = 45 \Rightarrow x_2 = -9$
(1) $\Rightarrow (-2)x_1 + (-63) + (-7) = (-66) \Rightarrow (-2)x_1 = 4 \Rightarrow x_1 = -2$

Lösung: $a = -2, b = -9, c = -1$

Viel Erfolg!